Yield Curve Modeling with Macro Factors

An Implementation of the Kim Filter in a Two-Economy Markov Regime Switching State-Space Model

András Bebes, Dávid Tran, László Bebesi

Abstract. We present a two-economy yield curve model with macro factors. We assume two distinct economic regimes and use a Markov regime switching state-space model to create a 5-year forecast for the Hungarian and Euro area factors. A rotated dynamic Nelson-Siegel model is used for yield curve modeling, we apply Gibbs sampling for parameter estimation and implement the Kim filter to determine the factor values.

Keywords: Yield Curve Modeling, Dynamic Nelson-Siegel Model, Markov Regime Switching, Gibbs Sampling

1 Introduction

In this paper we construct a Markov regime switching state-space model. We use this model to simultaneously forecast the Hungarian and the Euro area term structures of interest rates as well as some macroeconomic and financial factors, namely the Hungarian and Euro area inflation (CPI) rates, the Hungarian credit default swap (CDS) curve and the EUR/HUF exchange rate. The two yield curves and the macro factors are adequate for modeling the future cash-flows of outstanding government debt securities as well as instruments to be issued in the future by the Hungarian state. Forecasting cash-flows allows the debt manager to optimize the debt portfolio and conduct various scenario analyses.

The scope of this study extends to the model specification, parameter estimation and examination of the forecasted yield curves and macro-financial factors. Pricing the outstanding and future debt instruments as well as portfolio optimization is outside the scope of this paper.

Modeling and forecasting the term structure of interest rates (yield curve) is a well-researched topic. The most popular class of models used in empirical applications are variants of the Nelson-Siegel [1] model. A dynamic extension of the original static model was proposed by Diebold and Rudebusch [2]. We model the yield curve using a rotated dynamic Nelson-Siegel Model following the work of Nyholm [3].

State-space models are useful for modeling dynamic linear systems driven by unobservable parameters. These parameters can be estimated using the Kalman filter [4]. In practice, however, economic applications often exhibit nonlinearity. Hamilton [5] was the first to apply Markov regime switching models to economic problems with the purpose of dealing with time varying parameters. The unobservable parameters of Markov

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regime switching models can be estimated using an extension of the Kalman filter by Kim [6]. Our work follows Kim and Nelson [7] in using Gibbs sampling for parameter estimation and the Kim filter for determining the factor values.

Using a regime switching model is easily justified by economic considerations. Economic cycles are prevalent and can be separated into longer periods of growth (expansion) and shorter crisis periods of contraction (recession). These two distinct empirical situations allow for a meaningful application of time varying parameters. The steady state values (long-run expectations) determining the factors, the transition matrix and the error terms are all different based on the given regime. The heteroscedasticity of the error terms of the transition equation driving the factors can be handled using a different covariance matrix for every state.

Using a two-economy model with additional macro factors is also necessary as the Hungarian debt portfolio includes foreign currency denominated bonds as well as inflation-linked securities. Although the portfolio includes securities denominated in several currencies, the entire foreign currency exposure is only Euro denominated due to cross-currency swaps.

We can also model the dynamics between a small open economy (Hungary) and a large economy (the Euro area). Our model allows factor dynamics to be specified in a way that the Euro area factors can influence the Hungarian factors but not vice versa.

The structure of this paper is as follows: Section 2 deals with the specification of the used Markov regime switching model. Section 3 gives an overview of the process of parameter estimation. Section 4 introduces the data used for fitting the model. Section 5 gives an analysis of the forecast results. Section 6 concludes.

2 The Markov Regime Switching State-Space Model

In this section we construct a Markov regime switching state-space model that can be used to model the entirety of the Hungarian government debt portfolio and explore the relationship between the examined factors in a coherent and realistic way.

We use a 3-Factor Nelson-Siegel Model to estimate the HUF yield curve, while we assume that the Hungarian Euro yield curve at a specific maturity τ is the sum of the risk-free Euro yield curve and the Hungarian Credit Default Swap curve at maturity τ. We use a 3-Factor Nelson-Siegel Model to estimate the risk-free Euro yield curve.

Following Nyholm [3] we rotate the level and slope factors of the Nelson-Siegel Model to get a more transparent structure for our factors. Thus our level parameter reflects the level of the short-term rate, and the slope factor is positive in case of a normal (upward sloping) yield curve. In accordance with our assumptions the HUF ($y_{t,H}(\tau)$) and the risk-free Euro ($y_{t,E}(\tau)$) yield curve at time t and maturity τ is given by

$$y_{t,H}(\tau) = L_{t,H} + S_{t,H}\left(1 - \frac{1-e^{-\tau/\lambda_{t,H}}}{\tau/\lambda_{t,H}}\right) + C_{t,H}\left(\frac{1-e^{-\tau/\lambda_{t,H}}}{\tau/\lambda_{t,H}} - e^{-\tau/\lambda_{t,H}}\right) + \varepsilon_{\tau,H}, \tag{1}$$

$$y_{t,E}(\tau) = L_{t,E} + S_{t,E}\left(1 - \frac{1-e^{-\tau/\lambda_{t,E}}}{\tau/\lambda_{t,E}}\right) + C_{t,E}\left(\frac{1-e^{-\tau/\lambda_{t,E}}}{\tau/\lambda_{t,E}} - e^{-\tau/\lambda_{t,E}}\right) + \varepsilon_{\tau,E}, \tag{2}$$

where $\varepsilon_{\tau,H} \sim N(0, \quad \sigma^2_{\tau,H})$ and $\varepsilon_{\tau,E} \sim N(0, \quad \sigma^2_{\tau,E})$, $t = 1, \dots, n$. One can easily see that $\lim_{\tau \to 0} y_{t,k}(\tau) = L_{t,k}$ and $\lim_{\tau \to \infty} y_{t,k}(\tau) = L_{t,k} + S_{t,k}$ for $k \in \{H, E\}$. The curvature factor remains unchanged.

To avoid negative CDS prices we model the natural logarithm of the CDS curve with a 2-Factor Nelson-Siegel Model. The Hungarian CDS curve $(y_{t,CDS}(\tau))$ at time t and maturity τ is given by

$$\log\left(y_{t,CDS}(\tau)\right) = L_{t,CDS} + S_{t,CDS}\left(\frac{1-e^{-\tau/\lambda_{t,CDS}}}{\tau/\lambda_{t,CDS}}\right) + \varepsilon_{\tau,CDS}, \tag{3}$$

where $\varepsilon_{\tau,CDS} \sim N\left(0, \quad \sigma^2_{\tau,CDS}\right)$, $t = 1, \dots, n$.

Given our expectations of the system we also incorporate the Hungarian and Euro-zone inflation rates as well as the EUR/HUF exchange rate into our model. The latter is required to enable us to convert forecasted Euro cash-flows into a HUF basis. We observe the macro factors present in our model with measurement error, thus

$$mac_{t,j} = \widehat{mac}_{t,j} + \varepsilon_{mac_j}, \tag{4}$$

where $\varepsilon_{mac_j} \sim N\left(0, \quad \sigma^2_{j,mac}\right)$, $j \in \{cpi_{hun}, cpi_{eu}, eur/huf\}$. Given equations (1)-(4) we have a total of 11 factors.

Let $Y_t = \left[y_{t,H}, y_{t,E}, mac_{t,cpi_{hun}}, mac_{t,\mathrm{cpi}_{eu}}, \Delta\log\left(mac_{t,eur/huf}\right), \log\left(y_{t,CDS}\right)\right]'$ be the vector of our observations[1] while $X_t = \left[L_{t,H}, S_{t,H}, C_{t,H}, L_{t,E}, S_{t,E}, C_{t,E}, mac_t, L_{t,CDS}, S_{t,CDS}\right]$ is the vector of unobserved state variables i.e. the factors at time t $(t = 1, \dots, n)$.

Let S_t denote an unobserved, discrete-valued 2-state Markov-switching variable $(S_t \in \{1,2\})$ reflecting our prior assumption that the economy switches between two distinct regimes. The first regime (normal regime) represents economic stability and growth, while the second one (crisis regime) is connected to a more turbulent crisis situation.

The transition probabilities of the 2-state Markov process are given by

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}, \tag{5}$$

where $p_{ij} = Pr(S_t = j | S_t = i)$ and $p_{i1} + p_{i2} = 1$ for all $i \in \{1,2\}$.

The state-space representation of our dynamic linear model at time t can be written as:

$$Y_t = H_t X_t' + \theta_t, \tag{6}$$

[1] Both $y_{t,H}$ and $y_{t,E}$ contain maturities for $\tau \in \{1M, 2M, 3M, 6M, 9M, 12M, 2Y, 3Y, 4Y, 5Y, 7Y, 10Y, 12Y\}$, while $y_{t,CDS}$ contains maturities for $\tau \in \{6M, 12M, 2Y, 3Y, 4Y, 5Y, 7Y, 10Y\}$.

$$X_{t+1} = \mu_{S_t} + X_t F_{S_t} + \varepsilon_t, \tag{7}$$

$$\begin{bmatrix} \theta_t \\ \varepsilon_t' \end{bmatrix} \sim N\left(0, \quad \begin{pmatrix} R_{S_t} & 0 \\ 0 & Q_{S_t} \end{pmatrix}\right) \tag{8}$$

with $t = 1, \ldots, n$. The measurement equation (6) describes the relation between the data and our factors, where Y_t, θ_t are $m \times 1$ vectors, X_t, ε_t are $1 \times f_n$ vectors, m determines the dimension of the measurement variables, while f_n accounts for the number of factors in the model[2]. H_t is an $m \times f_n$ matrix that links the observed variables to the unobserved factors at time t, mainly containing the factor-loadings of equations (1)-(3). H_t does not depend on the Markov variable S_t, it is only dependent on the time-decay parameters, $\lambda_{t,k}$ $k \in \{H, E, CDS\}$.

The transition equation (7) describes the dynamics of the state variables at time t, where F_{S_t} $(f_n \times f_n)$ is the transition matrix describing the evolution of our factors, while μ_{S_t} represents the drift component in the transition equation, both are dependent on the Markov-switching variable S_t. R and Q are covariance matrices with appropriate dimensions. The (i,j)-th element of the parameters dependent on S_t can be described by the equation

$$f_{i,j,S_t}^k = f_{i,j,1}^k S_{1t} + f_{i,j,2}^k S_{2t}, \tag{9}$$

where k denotes the chosen parameter ($k \in \{\mu, F, R, Q\}$), with $S_{lt} = 1$ if $S_t = l$, and $S_{lt} = 0$ otherwise for $l \in \{1,2\}$.

Let C be the vector of the steady state factor values, given by $C_{S_t} = \mu_{S_t}\left(I_{f_n} - F_{S_t}\right)^{-1}$, where I_{f_n} is the identity matrix with dimensions $(f_n \times f_n)$. Extracting it from equation (7) and denoting $X_t^* = X_t - C_{S_t}$ we get $X_t^* = X_t^* F_{S_t} + \varepsilon_t$, which we can use to generate samples for C in each step of the Gibbs sampler from the posterior distribution suggested by Villani [8], and thus we can replace μ from our parameter set. Considering that the factors are easy to interpret in econometrics, the steady state values hold valuable information. While an analyst does not necessarily have expectations for a given element of the drift parameter μ, they may have one for the Eurozone CPI.

We have two additional restrictions based on our assumptions. First, R_{S_t} is diagonal, thus at every time t ($t = 1, \ldots, n$) the estimation error in our yield curve points and the measurement errors in the macro factors are uncorrelated. Second, we assume that the foreign factors can influence the domestic ones but not vice versa.

3 Parameter Estimation

This section deals with the problem of estimating the parameters of the system described in Section 2. We call the parameters H, R, C, F and Q the model's hyperparameters, while the entirety of our parameter space is denoted by $\Theta \in$

[2] The index in f_n is in no connection with the count of the observations; n.

$\{H_j, F_i, C_i, Q_i, R_i, \Pr, S_j\}$ ($i \in \{1, 2\}, j \in (1, \dots, n)$). To estimate the values of the factors we turn to the work of Kim [6], while for the random number generation of the models hyperparameters we use the distribution suggested by Villani [8] and Sugita [9] within the framework of a Gibbs sampler.

3.1 Kim Filter

The Kim filter is an extension of the Kalman filter [4] which gives a solution to the parameter estimation problem of general dynamic linear models with a Markov-switching parameter put in a state-space representation form.

Suppose that the hyperparameters of the model defined in Section 2 are known. Let $\mathcal{F}_t$ be the information available at time t. As defined in the Kalman filter the goal is to forecast X_t based on information available at time $t-1$. Let

$$X_{t|t-1}^{(i,j)} = E(X_t | \mathcal{F}_{t-1}, S_t = j, S_{t-1} = i) \tag{10}$$

be the conditional expectation of X_t based on $\mathcal{F}_{t-1}$ and $S_t = j, S_{t-1} = i$, and

$$P_{t|t-1}^{(i,j)} = E\left(\left(X_t - X_{t|t-1}^{(i,j)}\right)' \left(X_t - X_{t|t-1}^{(i,j)}\right) \middle| \mathcal{F}_{t-1}, S_t = j, S_{t-1} = i\right) \tag{11}$$

the mean squared error of the forecast. Then, conditional on $S_t = j, S_{t-1} = i$ at time t, the following 7 equations define the Kalman filter algorithm; the first 4 define the phase of prediction, the last 2 the phase of updating, while the fifth defines the Kalman gain:

$$X_{t|t-1}^{(i,j)} = \mu_j + X_{t-1|t-1}^{(i)} F_j, \tag{12}$$

$$P_{t|t-1}^{(i,j)} = F_j' P_{t-1|t-1}^{(i)} F_j + Q_j, \tag{13}$$

$$\eta_{t|t-1}^{(i,j)} = Y_t - H_t \left(X_{t|t-1}^{(i,j)}\right)', \tag{14}$$

$$f_{t|t-1}^{(i,j)} = H_t P_{t|t-1}^{(i,j)} H_t' + R_j, \tag{15}$$

$$K_t^{(i,j)} = P_{t|t-1}^{(i,j)} H_t' \left(f_{t|t-1}^{(i,j)}\right)^{-1}, \tag{16}$$

$$X_{t|t}^{(i,j)} = X_{t|t-1}^{(i,j)} + \left(K_t^{(i,j)} \left(\eta_{t|t-1}^{(i,j)}\right)'\right)', \tag{17}$$

$$P_{t|t}^{(i,j)} = \left(I_{fn} - K_t^{(i,j)} H_t\right) P_{t|t-1}^{(i,j)}. \tag{18}$$

$X_{t|t}^{(i)} = E(X_t | \mathcal{F}_t, S_t = i)$ is the conditional expectation of X_t based on $\mathcal{F}_t$ and $S_t = i$, while $P_{t|t}^{(i)} = E\left(\left(X_t - X_{t|t}^{(i)}\right)' \left(X_t - X_{t|t}^{(i)}\right) \middle| \mathcal{F}_t, S_t = i\right)$ is the mean squared error of X_t conditional on $\mathcal{F}_t$ and $S_t = i$. $\eta_{t|t-1}^{(i,j)}$ is the conditional forecast error of Y_t based on $\mathcal{F}_{t-1}$ and $S_{t-1} = i$, $S_t = j$, while $f_{t|t-1}^{(i,j)}$ is the conditional covariance of $\eta_{t|t-1}^{(i,j)}$.

Assuming that X_t is stationary, the unconditional mean and covariance matrix and the steady state probabilities can be used to start the filter.

To handle the increase in the number of cases at each iteration caused by the different states, Kim suggests the following equations to collapse the terms $X_{t|t}^{(i,j)}$ and $P_{t|t}^{(i,j)}$ to $X_{t|t}^{(j)}$ and $P_{t|t}^{(j)}$ respectively

$$X_{t|t}^{(j)} = \frac{\sum_{i=1}^{M} \Pr(S_t = j, S_{t-1} = i|\mathcal{F}_t) X_{t|t}^{(i,j)}}{\Pr(S_t = j|\mathcal{F}_t)}, \tag{19}$$

$$P_{t|t}^{(j)} = \frac{\sum_{i=1}^{M} \Pr(S_t = j, S_{t-1} = i|\mathcal{F}_t)\left(P_{t|t}^{(i,j)} + \left(X_{t|t}^{(j)} - X_{t|t}^{(i,j)}\right)'\left(X_{t|t}^{(j)} - X_{t|t}^{(i,j)}\right)\right)}{\Pr(S_t = j|\mathcal{F}_t)}. \tag{20}$$

Using these approximations, the problem becomes feasible. To approximate the probability terms in equations (19) and (20) one can use the Hamilton filter suggested by Kim and Nelson [7], thus making the filter complete.

3.2 Estimation of The Model Hyperparameters

With the assumption that H, R, C, F and Q parameters are known for both regimes, the Kim filter is applicable to estimate the values of X_t, and the $P(S_t = j|\mathcal{F}_t)$ probabilities, while S_t can be acquired through random number generation for all $t = 1, \dots, n$. For the estimation of parameters R, C, F and Q for a given X we use Gibbs sampling.

Assuming that $vec(F_{prior})$ is multidimensional normal with $vec(F_0)$ mean and V_0 covariance matrix, while Q_{prior} is inverse-Wishart with Q_0 scale matrix and n_0 degrees of freedom, Sugita [9] has shown that

$$vec(F_{posterior})|Q, Y \sim N(vec(F^*), \quad V), \tag{21}$$

$$Q_{posterior}|F, Y \sim IW(Q^*, \quad n^*), \tag{22}$$

with $V = \left(V_0^{-1} + Q^{-1} \otimes (X'X)\right)^{-1}$ and $vec(F^*) = V\left(V_0^{-1} vec(F_0) + (Q^{-1} \otimes I_m)^{-1} vec(X'Y)\right)$, while $Q^* = (Y - XF)'(Y - XF) + Q_0$, $n^* = n + n_0$, where V and V_0 are covariance matrices with appropriate dimensions $vec(\cdot)$ denotes the vectorization function, $\otimes$ the Kronecker product.

Assuming that C_{prior} is multidimensional normal with c_0 mean and Σ_0 covariance matrix, Villani [8] has shown that the conditional distribution $C_{posterior}$ is also multidimensional normal

$$C_{posterior}|Q, F, X^* \sim N(c^*, \quad \Sigma^*), \tag{23}$$

with $\Sigma^* = (U'(D'D \otimes Q^{-1})U + \Sigma_0^{-1})^{-1}$, $c^* = \left(\Sigma^*(U' vec(Q^{-1}(X^*)'D) + \Sigma_0^{-1} c_0')\right)'$, where $\mathrm{U} = [I_m \quad F]'$, $D = [1_n \quad -1_n]$. 1_n is the vector of ones with dimension $n \times 1$, while I_m is the identity matrix with $m \times m$ dimensions.

With the proper transformations of distributions (21)-(23) we can generate samples for parameters R, C, F and Q in each step of the Gibbs sampler for both regimes. The selection of the starting covariance matrices V_0 and Σ_0 is crucial in a sense that it determines the parameter space explored.

We use the Random Walk Metropolis Hastings algorithm for the estimation of H. H_t is only dependent on $\lambda_{t,H}, \lambda_{t,E}, \lambda_{t,CDS}$. $\lambda_{t,k}$ is determined using the $\lambda_{t,k}^{new} = \lambda_{t,k} + \kappa_t \varepsilon_{t,k}$ equation, where $k \in \{H, E, CDS\}$, κ_t can be arbitrarily chosen and $\varepsilon_{t,k} \sim N(0,1)$ for every t and k. Using the generated parameter we calculate a candidate H, which we keep with a calculated α probability, after the evaluation of the measurement equation's likelihood function.

3.3 The Algorithm

Our parameter estimation algorithm is based on the works of Blake & Mumtaz [10]. The algorithm consists of the following 7 steps:

1. We choose appropriate starting points, which are in the parameter space. Our strategy is the following: we substitute the latent factors with their empirical ones. With these we estimate the system's transition equation using the OLS method. Based on the absolute error of the transition equation we organize and divide them into two groups. The empirical covariance matrices of these groups specify the starting covariance matrices. The Kim filter becomes applicable after the setting of the starting parameters.
2. Given the parameter space (Θ) of the model we run the Kim filter, thus we get the values of X_t for every time $t = 1, \dots, n$.
3. In the third step we use the $P(S_t = j|\mathcal{F}_t)$ probabilities acquired from the Kim filter to generate the probabilities of being in a given regime at time t starting from $t = n$ heading towards $t = 1$ using the transition probabilities. We determine the values of S_t for every $t = 1, \dots, n$ using random generation.
4. In the fourth step we generate new transition probabilities. Let π (1×2) be the vector of steady state probabilities, and P_{prior} the matrix of transition probabilities, then

   $$M_{prior} = P_{prior} \circ \left(1_2 \otimes (\pi n)\right)$$

   defines the prior state-transition matrix, with $\circ$ denoting the element-wise multiplication and $\otimes$ the Kronecker product. The empirical state-transition matrix M_{emp} comes from the realizations of S_t generated in step 3. Thus

   $$P_i^{new} = Beta\left(M_{i,emp} + M_{i,prior}\right)$$

   with i denoting the rows of the specified matrices ($i \in \{1,2\}$), while the Beta distribution comes as a special case of the Dirichlet distribution for $M = 2$ states[3].

[3] The general idea for the generation of transition probabilities follows the work of Kim and Nelson [7], whereas a detailed and intuitive description of the Dirichlet distribution is given by Frigyik, Kapila and Gupta [11].

5. Step five deals with the generation of the parameters Q, R, C, and F. We use random number generation based on the distributions suggested by Sugita [9] and Villani [8].
6. In step six we estimate H_t and $\lambda_{t,H}, \lambda_{t,E}, \lambda_{t,CDS}$ with the Random Walk Metropolis Hastings algorithm.
7. We repeat steps 2-6 until the convergence meets our expectations.

4 Data

In this section we give an overview of the macro-financial and yield curve data used for fitting the Markov regime switching model. We also address the challenges regarding the construction of yield curves and introduce our prior assumptions regarding the model. We use data with monthly frequency from October 2008 to December 2017 for a total of $n = 111$ observations for each time series. The limitation is due to the lack of earlier Hungarian CDS data. This gives us about 10 years of historical data and most importantly, includes the 2008 crisis and the following recession period, giving merit to using a regime switching model.

Our data sources include Thomson Reuters for CDS curve and Euro area yield curve points, the Hungarian Central Bureau of Statistics for Hungarian CPI data, Eurostat for the Euro area CPI data and the Hungarian National Bank for EUR/HUF exchange rate time series.

To obtain Hungarian yield curve data, we use the secondary market bid-ask quotes of HUF denominated Hungarian government bonds, as price quotes provided by primary dealers are considered as actual market prices, the fixing process of which is regulated by the Hungarian Government Debt Management Agency.

We use the Fama-Bliss [12] method to transform price quotes into yield curve points upon which we can fit the Nelson-Siegel model. This nonparametric method allows us to obtain zero-coupon yield curve points from bond price quotes. The Nelson-Siegel model can then be fitted on the unsmoothed Fama-Bliss yields.

We impose additional restrictions compared to the original Fama-Bliss [12] algorithm, using only fixed interest rate bonds with a maturity over 1 year as well as zero-coupon T-bills. Our implementation of the Fama-Bliss [12] algorithm can be summarized in four steps:

1. We determine whether the Yield-to-Maturity (YTM) of a given security is between the moving average of the previous 3 and the next 3 (based on time to maturity) government securities' yields.
2. We examine the unsmoothed Fama-Bliss forward yields. We exclude a government security from the analysis if the forward rate corresponding to its maturity shows a change greater than 0.2 percentage points in the opposite direction. In this step we calculate the forward rates using only government securities passing step 1.
3. We calculate the forward yield curve once more. Using the remaining government securities after step 2, we calculate the moving averages as in step 1. An instrument is to be considered in the analysis if the forward rate corresponding to its maturity is

between the two moving averages or the difference from one of them is not larger than 0.2 percentage points.

4. We repeat steps 2 and 3 using instruments excluded from the analysis but fulfilling the initial conditions.

In the end, we get a forward yield curve that fits the two smoothness criteria defined in steps 2 and 3. The forward yield curve can be then transformed into a zero-coupon yield curve.

Fitting a Markov regime switching model requires us to give an initial estimation of the transition probability matrix (Eq. 5). We set the starting value to:

$$P_{prior} = \begin{pmatrix} 0.987 & 0.013 \\ 0.180 & 0.820 \end{pmatrix}.$$

This means that the probability of going from the first regime to the second would be 1.3%, while exiting from the second regime would be 18%. Thus, the first regime is expected to last 6.5 years, whereas the expected duration of the second regime is only half a year. The prior steady state values of the factors were determined using P_{prior} and incorporate the effects of the current historically low interest rate period.

The Hungarian CPI expectation in the first regime equals the inflation target of the Hungarian Central Bank ($cpi_{prior,HUN} = 3\%$), and is one percentage point higher in the second regime. We expect the inflation rate of the Euro area to remain below the target set by the European Central Bank ($cpi_{prior,EU} = 1.5\%$). Regarding the HUF yield curve, we expect a 50bp increase in the short rate compared to 2017 December, and a 3.5% level for the long end of the yield curve. Our prognosis for the yield curve also includes a flattening slope and a rising level in case of a crisis.

Finally, we expect the EUR/HUF exchange rate to stagnate during the first regime and rise during the second regime, meaning a comparatively weaker HUF and stronger Euro in a crisis situation.

5 Results

This section is split into two parts. First, we give an overview of the results of the fitted model, including regime probabilities and steady state values. Finally, we make a 5-year forecast from end-2017 to end-2022 with 1 000 scenarios and examine its results[4].

5.1 Results of Model Fitting

The resulting transition probability matrix is somewhat different compared to the initial values:

$$P_{posterior} = \begin{pmatrix} 0.95 & 0.05 \\ 0.20 & 0.80 \end{pmatrix}.$$

[4] The length of the forecast aligns with the medium term debt management framework of the Hungarian Government Debt Management Agency.

According to the posterior probabilities, the normal regime lasts for two years on average while a typical crisis lasts for 5 months.

Regime	L_H	S_H	C_H	L_E	S_E	C_E	cpi_H	cpi_E	EUR/HUF	L_{CDS}	S_{CDS}
Normal	0.40	3.22	-3.10	-0.17	1.21	-1.15	2.79	1.36	-0.003	0.35	-2.64
Crisis	2.92	1.31	-0.10	0.28	1.03	-0.66	3.96	1.58	-0.011	0.97	-1.53

Table 1. Steady state factor values by regime

Table 1 shows the resulting steady state values. The Hungarian short rate is 0.4% in the normal regime and about 3% during a crisis, while the long rates are 3.6% and 4.2%, respectively. Steady state values for Hungarian inflation are 2.8% in the normal and 4% in the crisis regime.

Euro area short rates are close to zero in both regimes, while long rate steady state values are over 1%. Euro area inflation rates are consistent with our assumptions and do not reach the 2% ECB goal in any regime. The EUR/HUF exchange rate steady state values show the HUF getting slightly weaker compared to the Euro in the normal regime and weaker during a crisis.

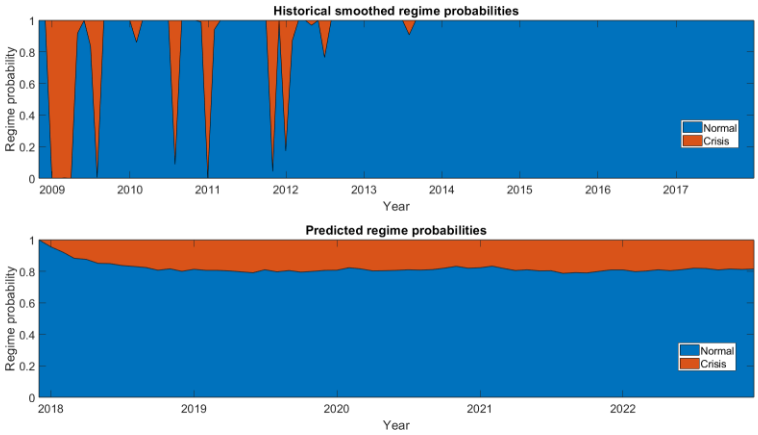

Fig. 1. Historical (upper panel) and predicted (lower panel) regime probabilities

Figure 1 shows the historical and predicted regime probabilities. Historical crisis occurrences can be easily connected to the recession period and the Greek debt crisis. The Hungarian economy has been relatively stable since 2014, which is also shown by the dominance of the normal regime since then. We can also see that from 2018 onwards we have an approximately 20% probability of slipping into the crisis regime, which is consistent with the estimated transition probabilities.

Although our experience suggests that using a regime switching model with the parameters described in this paper improves the robustness of the model, a quantitative proof is out of the scope of this study.

We have used 75 000 runs for the Gibbs sampling method. We discard the first 20 000 runs to alleviate the dependence on manually given prior values. Geweke test statistics [13] show that the simulation converges and 75 000 runs are enough for sampling.

5.2 Forecast Results

In order to make our predictions, we substitute the parameters gained from fitting the model into the transition equation. We generate 1 000 scenarios, all consisting of the trajectories of the yield curve and other macro-financial factors. The forecast horizon is 5 years with a monthly frequency.

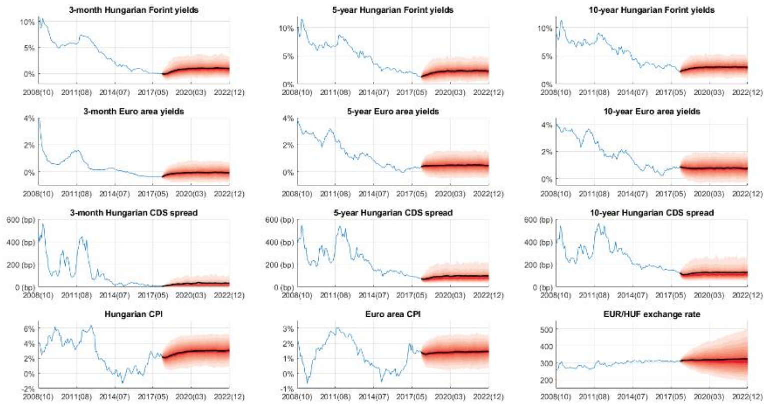

Fig. 2. Fan charts of selected yields, CDS spreads, CPI rates and exchange rate with 5-95% confidence intervals

Figure 2 shows the historical and predicted values of several yield curve points, CDS spreads and macro variables. We can see a gradual increase in Hungarian yield levels. By 2020, we predict a 1% short rate and a 3% 10-year yield. According to our forecast, the Euro area yields are mostly expected to stagnate with the short rate slowly converging to zero. Hungarian CDS spreads are expected to rise by $10-20$ basis points. The HUF shows a slight depreciation compared to the Euro. The fan charts also show that risks leading to the depreciation of the HUF as well as risks of higher interest rates are significantly higher than vice versa. CDS spreads in particular are especially vulnerable and can top 500bps in a crisis situation.

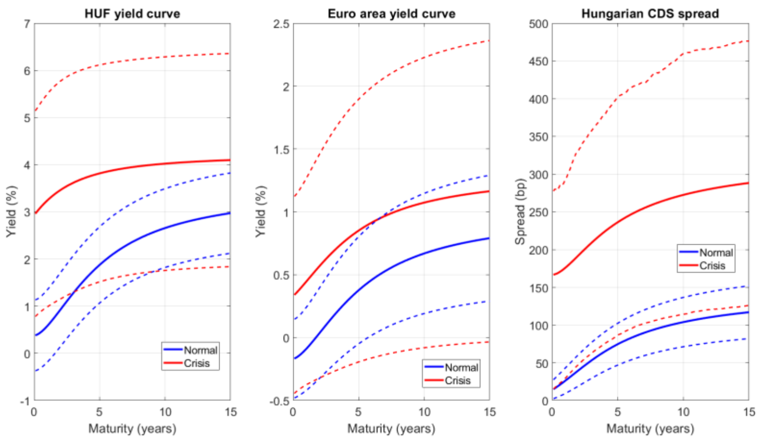

Fig. 3. Comparison of normal and crisis regime yield curves and CDS curves

Figure 3 shows that the two regimes are significantly different. The dashed lines show the standard deviation for the yield curves and the 16th and 84th percentile for the Hungarian CDS spread. They indicate a higher instability during the crisis regime for the CDS spread and both yield curves. The uncertainty of the Euro area yield curve is lower than the HUF yield curve during both regimes showing the inherent vulnerability of a small open economy compared to a large one. Yield curve and CDS levels are generally higher in the crisis regime. The Hungarian yield curve is also distinctly flatter in the second regime.

6 Conclusion

In this paper we used a Markov regime switching state-space model to forecast the Hungarian and Euro area yield curves with additional macro-financial factors on a 5-year horizon. We used a rotated Nelson-Siegel factor model for yield curve modeling and implemented the Kim filter to estimate the unobservable parameters. The fitted model matches our economic assumptions regarding normal and crisis regimes. We used data from October 2008 to December 2017. As of May 2018, the actual evolution of the factors is well within the confidence bounds of our predictions.

Further studies can be conducted to improve both the depth and the breadth of this analysis. A deeper analysis could include several competing models. Comparing and testing them using out-of-sample tests could highlight the different strengths of various models. A broader analysis could use the results of the forecast to predict the cash-flows of Hungarian government debt securities and calculate the costs of these together with the distribution of the costs, which is one of the risk factors that public debt managers face. Thus, a cost-risk optimization or a scenario analysis can be carried out on the Hungarian debt portfolio.

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